Extra Credit 12

Given a chessboard with 4 rows and 3 columns, 3 white knights (horses) are in the top row and 3 black in the bottom row.

In the least number of moves exchange the horses. Hint: use graph representation of the problem.

B   B   B                            W  W  W

0   0   0     min # of turns    0   0   0

0   0   0    =======>     0   0   0

W W  W                             B   B   B

Minimum no. of moves = 16.

This problem can be solved by using the theory of planar graphs.

The squares of the chessboards represent nodes of a graph and the possible moves of the pieces between the corresponding squares (the nodes of the graph) are interpreted as the connecting lines of the graph.

The corresponding graph for the board and the knights is shown in figure:-

Note: The initial positions of the knights are indicated, and all possible moves of the knights are between the squares (the nodes of the graph) are marked by lines.

A picture containing radar chart

Description automatically generated



Using Dudeney’s method of unraveling a graph, starting from any node, the above graph can be reduced to the equivalent graph shown, which is much clearer and more convenient for the analysis.



The topological order



and connectedness



are preserved.



To find the solution, we have to move the knights along the circumference graph until they change places.

1-6-7, 11-6-1, 3-4-11, 10-9-4-3, 2-9-10, 7-6, 12-7-2, 6-7-12.

So, the minimum number of moves is 16.